Introduction to Statistics Tutorial: Large-Sample Test of Hypothesis

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2008

Outline

- Four parts of statistical hypothesis testing
- One- and Two-tailed tests
- Type I and II Errors
- Power of the test
- Step by Step Procedure for a Large-Sample Test of Hypothesis
- Example: Large-Sample Test of Hypothesis about a Population Mean
- Conclusion: Generalization

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Introduction

A **hypothesis** in statistics, is a claim or statement about a property of a population.

A statistical test of hypothesis consists of four parts:

1. A null hypothesis (the questioned hypothesis)
2. An alternative hypothesis (the hypothesis the researcher wishes to support)
3. A test statistic
4. A rejection region

Null Hypothesis, H_o

The null hypothesis is a statement about the value of a population parameter

The null hypothesis contains a condition of equality: =, ≥, or ≤

Test the Null Hypothesis directly

Results: Reject H_0 or fail to reject H_0

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Alternative Hypothesis, H_a

Hypothesis the researchers wishes to support

Must be true if H_0 is false

Contains $\neq$, $<$, $>$

Opposite of the Null

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Test Statistic

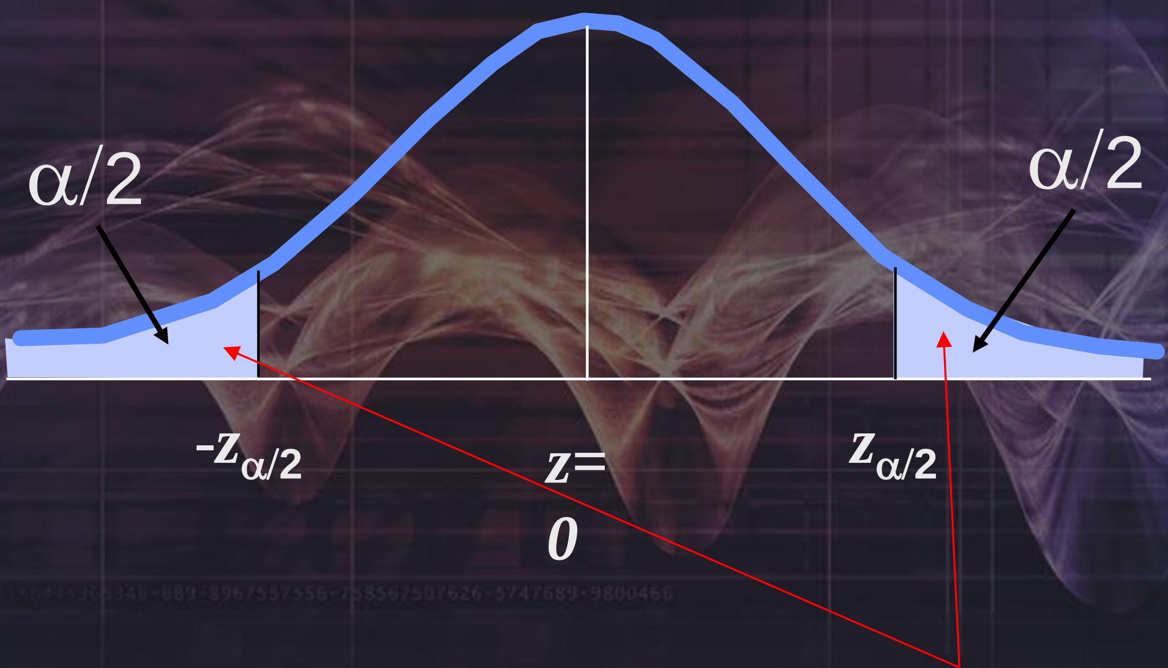
A value computed from the sample data that is used in making the decision about the rejection of the null hypothesis

For large samples, testing claims about population means:

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}}$$

Rejection Region

Set of all values of the test statistic that would cause a rejection of the null hypothesis



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Significance Level, α

The probability that the test statistic will fall in the critical region when the null hypothesis is actually true.

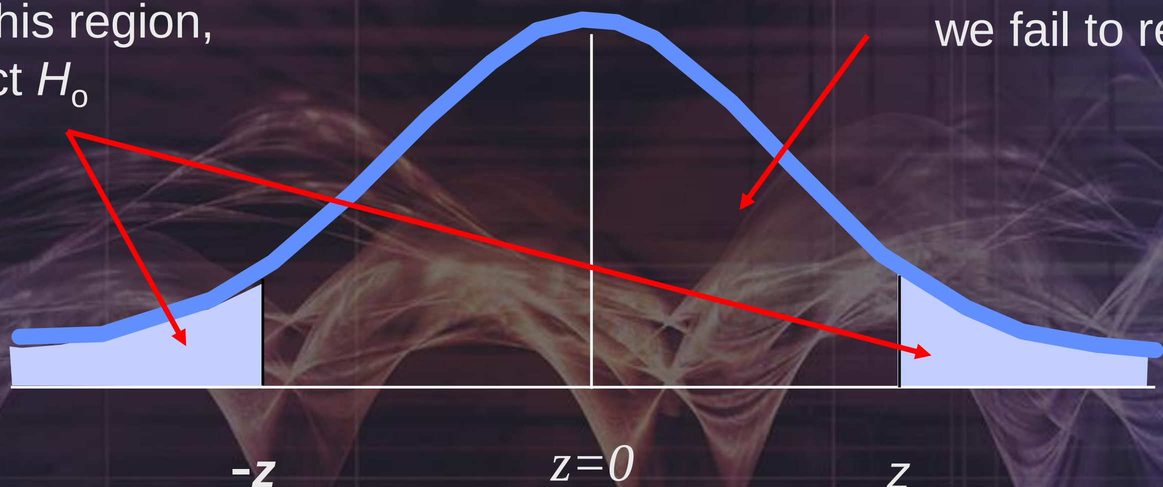
Common choices are 0.05, 0.01, and 0.10

Same as degree of confidence for a confidence interval

The Critical Value: z

If the test statistic falls in this region, we reject H_o

If the test statistic falls in this region, we fail to reject H_o

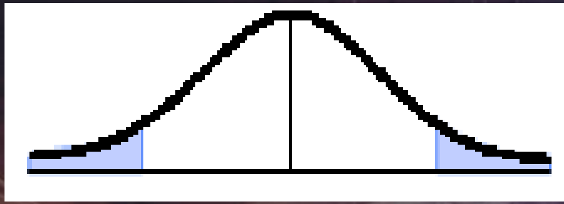


Value or values that separate the critical region (where we reject the null hypothesis) from the values of the test statistics that do not lead to a rejection of the null hypothesis

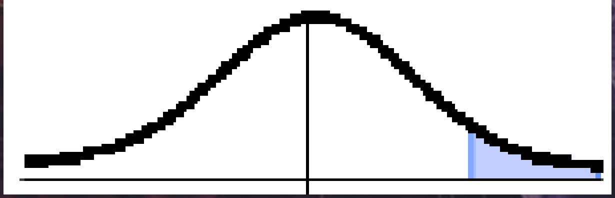
Rejection regions

The tails in a distribution are the extreme regions bounded by critical values.

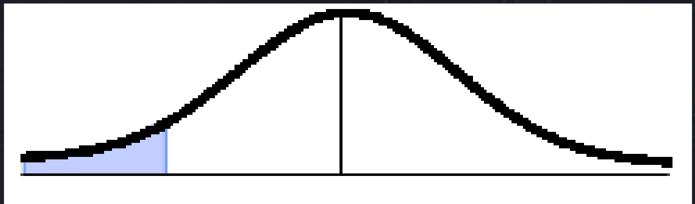
- Two-tailed



- Right-tailed



- Left-tailed



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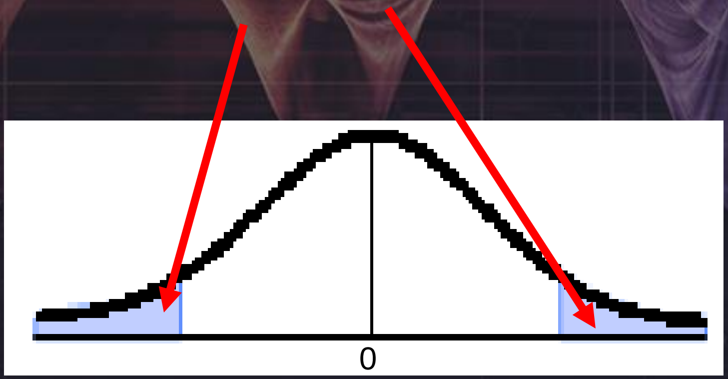
The Two-Tailed Test

H_0: μ = 15
H_a: μ ≠ 15

α is divided equally between the two tails of the critical region

If the test statistic falls in either one of the rejection regions (values that differ significantly from 15), then you reject the null hypothesis.



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The One-Tailed Test

(right-tail)

H_0: μ ≤ 15
H_a: μ > 15

α is the area in the right tail of the critical region

If the test statistic falls in right-tailed rejection region (values that are significantly greater than 15), then you reject the null hypothesis.

The One-Tailed Test

(left-tail)

H_0: μ ≥ 15
H_a: μ < 15

α is the area in the left tail of the critical region

If the test statistic falls in left-tailed rejection region (values that are significantly less than 15), then you reject the null hypothesis.

Conclusion Statements for Hypothesis Testing

We always test the null hypothesis.

1. Reject the H_0

2. Fail to reject the H_0

Final conclusions to a hypothesis test:

- we are not proving the null hypothesis
- sample evidence is not strong enough to warrant rejection of the null hypothesis
- some statisticians use “accept the null hypothesis”

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Type I Error

The error made by rejecting the null hypothesis when it is true.

α (alpha) is used to represent the probability of a type I error

Example: Rejecting a claim that the mean body temperature is 98.6 degrees when the mean really does equal 98.6

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Type II Error

The error made by failing to reject the null hypothesis when it is false.

ß (beta) is used to represent the probability of a type II error

Example: Failing to reject the claim that the mean body temperature is 102.6 degrees when the mean is really different from 102.6

Type I and Type II Errors

		Null Hypothesis	
		The null hypothesis is True	**The null hypothesis is False**
Decision	**We decide to reject the null hypothesis**	Type I Error (rejecting a true null hypothesis)	Correct decision
	We decide to fail to reject the null hypothesis	Correct decision	Type II Error (rejecting a false null hypothesis)

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Power of a Statistical Test

The power of a hypothesis test is the P[Reject the null hypothesis when the null hypothesis is false] = 1 - β

Measures the ability of the test to perform as required

Large-Sample Test of Hypothesis about a Population Mean

Assumptions:

1. sample is randomly selected
2. sample is large (n > 30) → CLM applies
3. If σ is unknown, we can use sample standard deviation S as estimate for σ.

Goal: Identify a sample result that is *significantly* different from the claimed value; in this case, is our sample mean statistically different from the claimed null hypothesis mean?

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Large-Sample Test of Hypothesis about a Population Mean

Step by Step

1. Identify the null hypothesis (specific claim to be tested)

$$H_0: \mu = \mu_0$$

2. Identify the alternative hypothesis that must be true when the original claim is false.

one-tailed test　　　two-tailed test

$H_a: \mu > \mu_0$　　*or*　　$H_a: \mu \neq \mu_0$

(or, $H_a: \mu < \mu_0$)

3. Calculate the test statistic:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

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Large-Sample Test of Hypothesis about a Population Mean

Step by Step

4. Select the significant level α based on the seriousness of a type I error. The values of 0.05 and 0.01 are very common.
5. Determine the critical values and the critical region. Draw a graph and include the test statistic, critical value(s), and critical (rejection) region.
6. Reject H_0 if the test statistic is in the critical region. Fail to reject H_0 if the test statistic is not in the critical region.
7. Restate this decision in simple, non-technical terms.

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Large-Sample Test of Hypothesis about a Population Mean

Example: Given a data set of 106 healthy body temperatures, where the mean was 98.2º and *s* = 0.62º , at the 0.05 significance level, test the claim that the mean body temperature of all healthy adults is equal to 98.6º. *(Example taken from Triola, Chapter 7, Elementary Statistics, Eighth Ed.)*

Steps 1 and 2: Identify the hypotheses.

$$H_0: \mu = 98.6$$

$$H_a: \mu \neq 98.6$$

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Large-Sample Test of Hypothesis about a Population Mean

Example: Given a data set of 106 healthy body temperatures, where the mean was 98.2º and *s* = 0.62º , at the 0.05 significance level, test the claim that the mean body temperature of all healthy adults is equal to 98.6º.

H_0: μ = 98.6

H_a: μ ≠ 98.6

Step 3: Calculate the test statistic:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{98.2 - 98.6}{\frac{0.62}{\sqrt{106}}} = \frac{-0.4}{0.06} = -6.64$$

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Large-Sample Test of Hypothesis about a Population Mean

Example: Given a data set of 106 healthy body temperatures, where the mean was 98.2º and *s* = 0.62º , at the 0.05 significance level, test the claim that the mean body temperature of all healthy adults is equal to 98.6º.

$$H_0: \mu = 98.6$$
$$H_a: \mu \neq 98.6$$
$$Z = -6.64$$

Step 4: Select the significance level α. This is given in our example to be 0.05.

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Large-Sample Test of Hypothesis about a Population Mean

H_0: μ = 98.6

H_a: μ ≠ 98.6

Z = -6.64

α = 0.05

Step 5: Draw a graph with critical values and the critical regions.

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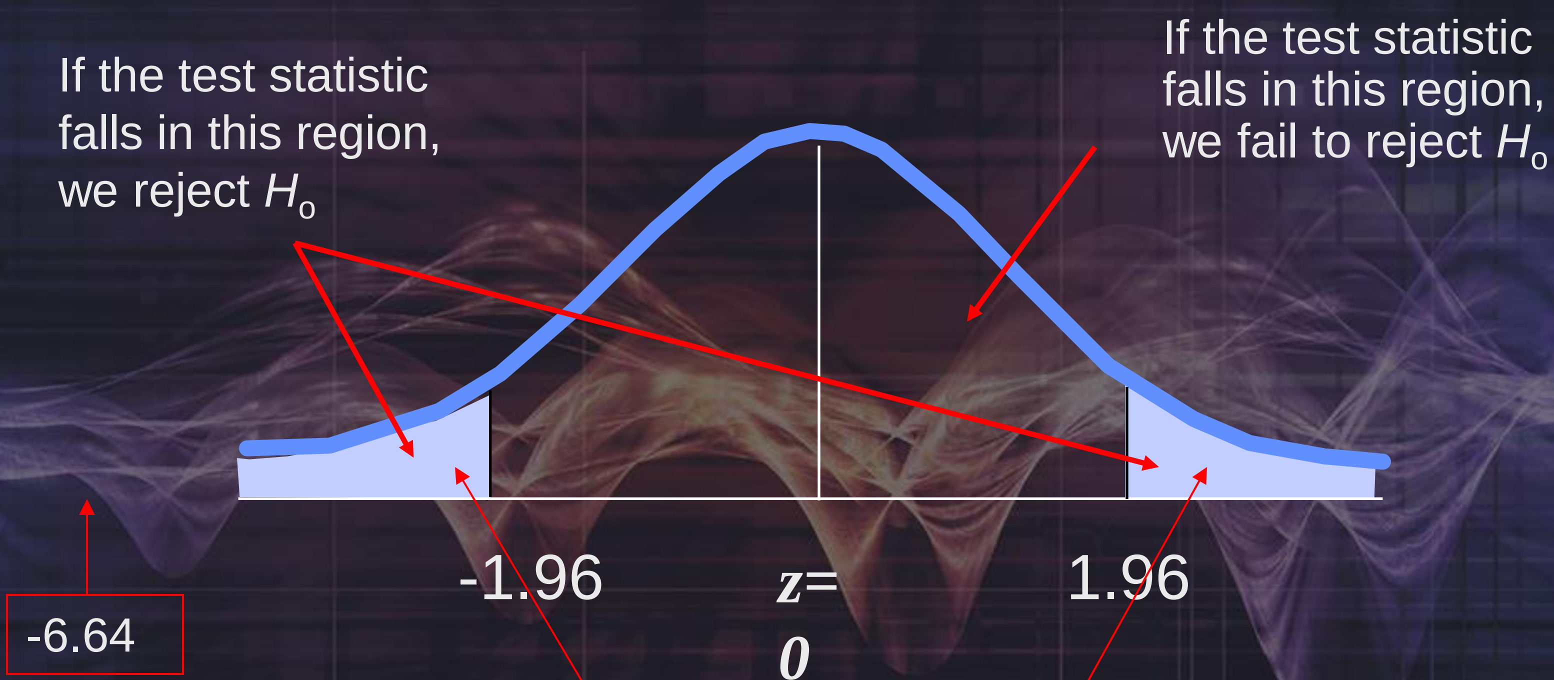
Large-Sample Test of Hypothesis about a Population Mean



Two tailed test – split the area between the two tails

$\alpha = 0.05$

$\alpha/2 = 0.025$

Use the normal table to find 1.96 as we did for the confidence intervals

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Large-Sample Test of Hypothesis about a Population Mean

Step 6: Reject H_0 since the test statistic z falls in the critical region in the left tail.

Step 7: Restate this decision in simple, non-technical terms:

There is sufficient evidence to warrant rejection of claim that the mean body temperatures of healthy adults is equal to 98.6º.

Large-Sample Test of Hypotheses Conclusion

Other tests of interest include:

- large-sample test for the difference between two populations means
- large-sample test for a binomial proportion
- large-sample test for the difference between two binomial proportions.

References

This tutorial is comprised of materials from the following sources:

Introduction to Probability and Statistics by Mendenhall and Beaver. ITP/Duxbury.

The Cartoon Guide to Statistics by Gonick and Smith. HarperCollins.

Basic Statistics: an abbreviated overview by Ackerman, Bartz, and Deville. 2006 Accountability Conference

Elementary Statistics, Eighth Ed. by Triola. Addison-Wesley-Longman. 2001

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