Introduction to Statistics Tutorial: Inference for Small Samples

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Outline

- Student's *t* Distribution
- Comparing the Student's *t* and Standard Normal Distributions
- Understanding the Student's t distribution
- Small sample inference concerning a population mean

Student's *t* Distribution (Properties)

1. The Student *t* distribution has the same general bell shape as the normal distribution; its wider shape reflects the greater variability that is expected with small samples.

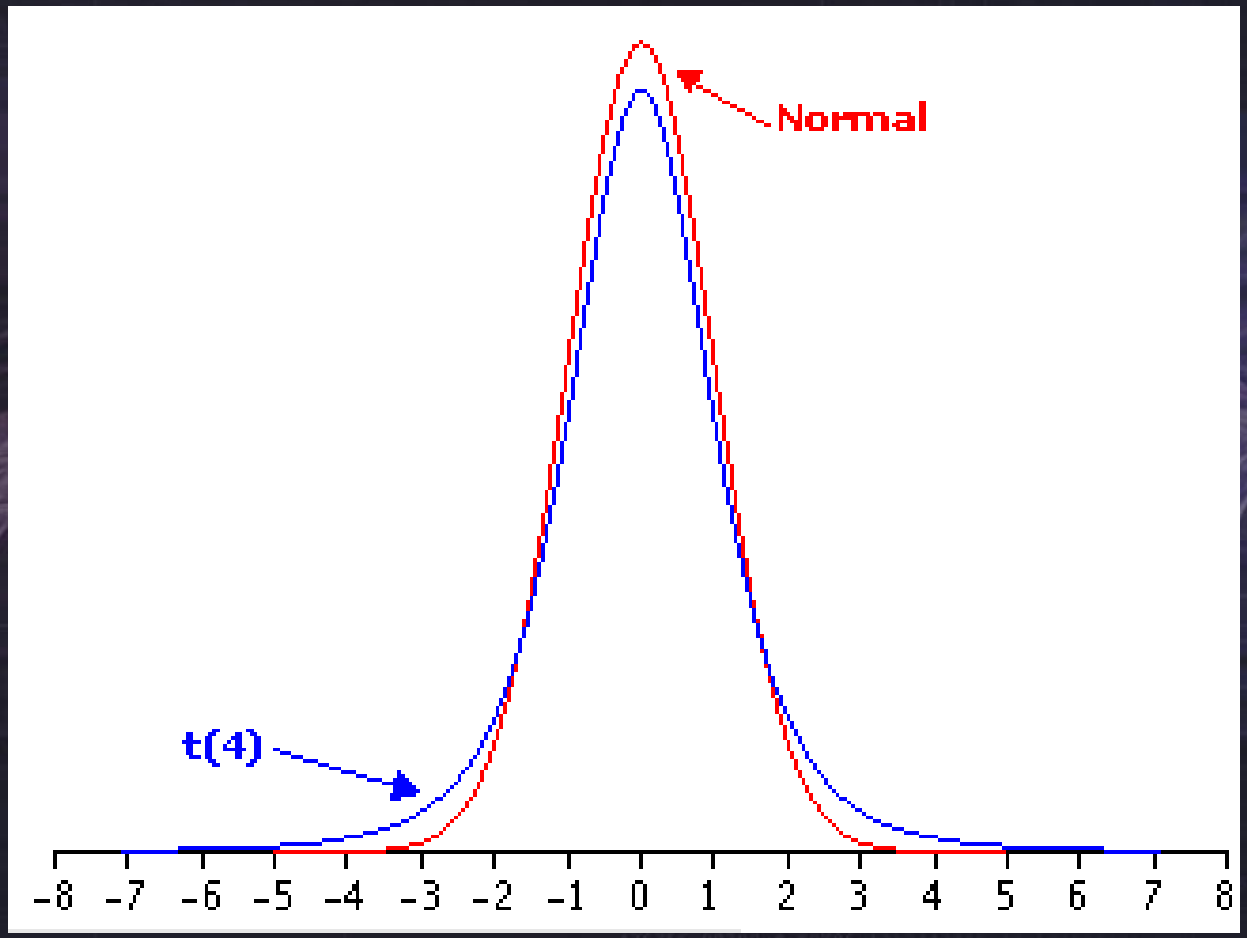
2. The Student *t* distribution is different for different sample sizes (based on the degrees of freedom)

3. The Student *t* distribution has a mean of $t = 0$ (similar to the standard normal distribution with a mean of $z = 0$).

4. The standard deviation of the Student *t* distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a $\sigma = 1$).

5. As the sample size *n* gets larger, the Student *t* distribution get closer to the normal distribution. For values of $n > 30$, the differences are so small that we can use the critical z values instead of the t distribution.

Student's *t* (*n*=6) vs. Standard Normal Distributions



Understanding Student's *t* distribution

Because of the thicker tails of the Student's *t* distribution, larger test statistics are needed to show significance.

The larger Student *t* critical value shows that with a small sample, the sample evidence must be more extreme before we consider the difference is significant.

Student's *t* Distribution

Assumptions for using the Student's t Distribution for testing claims about population means:

1. The sample is a simple random sample.
2. The sample is small ($n \leq 30$).
3. The value of the population standard deviation σ is unknown.
4. The sample values come from a population with a distribution that is approximately normal.

Test Statistic for a Student's *t* distribution

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Use the Student's t Distribution (not the Normal Table) Tables to find the critical values.

Need: Degrees of freedom (df) = n -1

Small sample inference concerning a population mean

Example: Given a data set of 11 healthy 5 year old children's weights where the mean was 43.5 pounds and s = 2.4 pounds, at the 0.05 significance level, test the claim that the mean weight of 5 year old children is equal to the average of 41.8 pounds listed in the medical books.

Identify the hypotheses and find the t test statistic:

H_0: μ = 41.8 vs. H_a: μ ≠ 41.8

n = 11 (*df = 11-1= 10*) α = 0.05

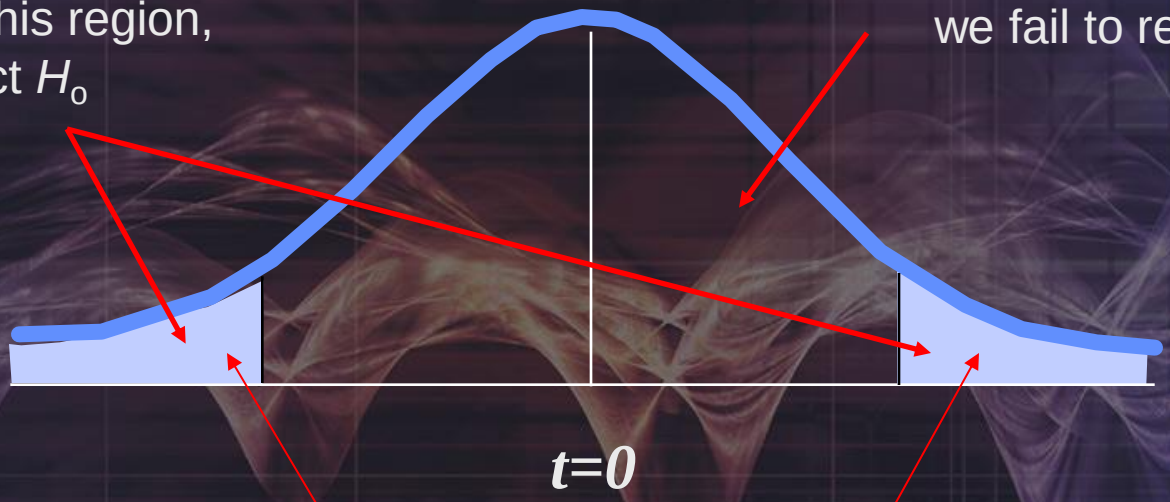
s = 2.4

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{43.5 - 41.8}{\frac{2.4}{\sqrt{11}}} = \frac{1.7}{0.72} = 2.36$$

Small-Sample Test of Hypothesis about a Population Mean

If the test statistic falls in this region, we reject H_o

If the test statistic falls in this region, we fail to reject H_o



Two tailed test – split the area between the two tails

α = 0.05

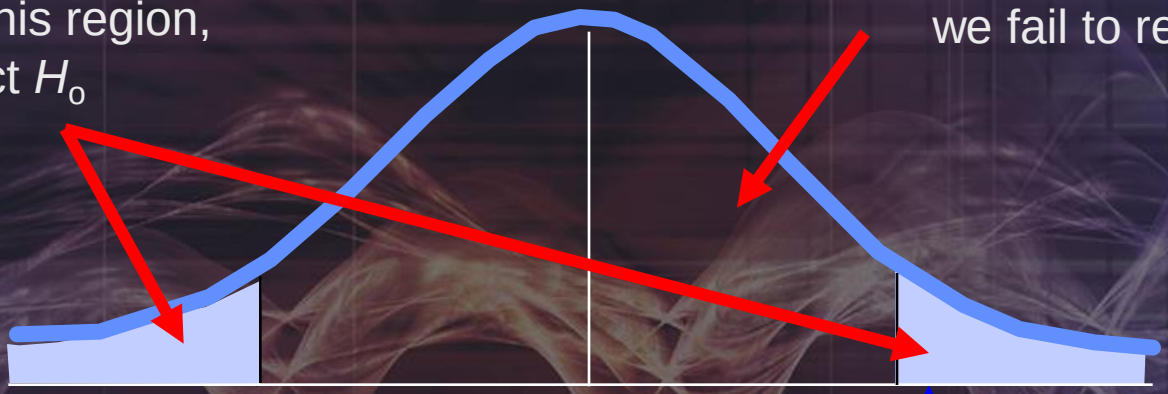
α/2= 0.025

Use the t distribution table to find the critical values

Small sample inference concerning a population mean

Tail Probabilities								
One Tail		0.10	0.05	0.025	0.01	0.005	0.001	0.0005
Two Tails		0.20	0.10	0.05	0.02	0.01	0.002	0.001
D	1	3.078	6.314	12.71	31.82	63.66	318.3	637
E	2	1.886	2.920	4.303	6.965	9.925	22.330	31.6
G	3	1.638	2.353	3.182	4.541	5.841	10.210	12.92
R	4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
E	5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
E	6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
S	7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
	8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
O	9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
F	10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
	11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
F	12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
R	13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
E	14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
E	15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
D	16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
O	17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
M	18	1.330	1.734	2.101	2.552	2.878	3.610	3.922

Small-Sample Test of Hypothesis about a Population Mean



Critical Values

Test Statistic = 2.36

Small-Sample Test of Hypothesis about a Population Mean

Since we rejected the null hypothesis, we conclude that *there is sufficient evidence to warrant rejection of the claim that the mean weight of 5 year olds is equal to 41.8 pounds.*

Small-Sample Test of Hypothesis

Other small-sample inference using Student's t Distribution for hypothesis testing include:

- The difference between two means
- A paired-difference of means
- Inferences about variance

References

This tutorial is comprised of materials from the following sources:

Introduction to Probability and Statistics by Mendenhall and Beaver. ITP/Duxbury.

Basic Statistics: an abbreviated overview by Ackerman, Bartz, and Deville. 2006 Accountability Conference

Elementary Statistics, Eighth Ed. by Triola. Addison-Wesley-Longman. 2001